possibly be used successfully as probes to investigate intensification of heat transfer in electric fields and enable much better methods of investigation to be developed, as well as simplifying the circuit and increasing the accuracy of measurements.

NOTATION

I_T, current through the thermistor, mA; U_T, thermistor voltage, V; U_{HV}, voltage of the high-voltage source, kV; Θ_T, thermistor temperature, °C; Θ_0 the ambient temperature, °C; $\Delta\Theta$, temperature drop, °C; P_T, power dissipated at the heating temperature Θ_T with no field, mW; ΔP, increase in power dissipated in a field at a temperature Θ_T, mW; k, dissipation factor of the thermistor, mV/°C; R_{20}, rated resistance of the thermistor at $\Theta_0 = 20°C$, kΩ; $E = U_{HV}\,[2.3\,r\log(R/r)]^{-1}$, electric field strength, kV/cm; R, inner radius of the cylinder, cm; r, radius of the thermistor, cm.

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DETERMINATION OF THERMOPHYSICAL CHARACTERISTICS OF HIGH HEAT-CONDUCTANCE MATERIALS ON THIN SAMPLES

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UDC 536.2.083

A method is proposed that makes it possible to eliminate the contact thermal resistances in the determination of the thermophysical characteristics of high heat conductance materials on samples with thickness $h \le 5 \cdot 10^{-3}$ m.

Thin samples of solid dielectrics with high heat conductance are widely used in radio equipment for space research. The thermophysical characteristics (t.p.c.) of the solid dielectrics has to be determined on thin samples. Such samples are easier to prepare without defects in the internal structure. Furthermore, samples of small thickness more closely correspond to real conditions of operation of solid dielectrics.

The thermophysical characteristics of very hard materials with thermal conductivity above 20 W/m · deg on samples of thickness less than $5 \cdot 10^{-3}$ m cannot be determined by the existing methods.

Stationary and many nonstationary methods require the introduction of the temperature measuring device into the investigated sample; this is difficult to accomplish in thin samples of great hardness. Moreover, in thin high heat conductance samples it is difficult to measure small time intervals of temperature increase due to rapid heat transfer.

Lensovet Leningrad Technological Institute, Leningrad. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 29, No. 6, pp. 1049-1056, December, 1975. Original article submitted November 1, 1974.

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The usual nonstationary methods that permit a complicated determination of t.p.c. are generally contact methods. The use of these methods for thermophysical investigations of high heat-conductance hard materials is not fruitful, since the magnitude of the contact heat resistance is of the same order as the heat resistance of the investigated samples.

In order to increase the accuracy of determination of t.p.c., usually the contact heat resistance is reduced by careful treatment of the contacting surfaces and use of lubricants with maximum thermal conductivity. However, a theoretical estimate of the effect of contact heat resistances [1, 2] on the accuracy in the determination of t.p.c. of high heat-conductance materials shows the inadequacy of the ordinary methods. Thus, for example, for a sample thickness $h = 4\text{-}5 \cdot 10^{-3}$ m and optical purity of the surface up to $1 \cdot 10^{-6}$ m the total thickness of Vaseline lubricant on four contacting surfaces will be $h_0 \simeq 4 \cdot 10^{-6}$ m. In this case the error in the determination of t.p.c. of polymethyl methacrylate, glass, bismuth, and silver by the method of time–temperature intervals will be:

1. Polymethyl methacrylate	$\Delta a/a = 0.3\%$;	$\Delta\lambda/\lambda = 0.2\%$.
2. Glass	— « — 2.0;	— « — 1.4.
3. Bismuth	— « — 12.6;	— « — 9.0.
4. Silver	— « — 89.5;	— « —63.0.

Therefore, the presence of contact heat resistances, their difference for different samples, and their nonconstancy for identical samples in different experiments constitute the main source of errors in the determination of t.p.c. of high heat-conductance materials. Even the most careful polishing and lubrication of the samples is not effective in eliminating these factors. In order to determine t.p.c. of high heat-conductance materials on thin samples it is necessary to develop methods that would be free from the disadvantages mentioned above.

For this purpose we have made use of certain characteristics of the temperature field, on which the method of two time–temperature intervals is based [3, 4], and of the law of additivity of heat resistances, the use of which is permissible in nonstationary regimes if the thermal activity of the heat receiver is greater than the thermal activity of the investigated material [5].

The temperature field of the investigated sheet, in contact with a semiinfinite cylinder playing the role of the heat receiver, is described by the equation [3]

$$\theta = (1 + \alpha)(\operatorname{erfc} y - \alpha \operatorname{erfc} 3y + \alpha^2 \operatorname{erfc} 5y - \ldots),$$

where

$$y = \frac{h}{2\sqrt{a\tau}}, \quad \alpha = \frac{\varepsilon - 1}{\varepsilon + 1}, \quad \varepsilon = \frac{\lambda}{b\sqrt{a}}, \; b = \frac{\lambda_B}{\sqrt{a_B}},$$

$$\operatorname{erfc} y = 1 - \operatorname{erf} y, \; \operatorname{erf} y = \frac{2}{\sqrt{\pi}} \int_0^y e^{-y^2} dy. \tag{1}$$

We note that the direct application of the method of two time–temperature intervals [3] for determining t.p.c. in conditions $\varepsilon < 1$ and $\Theta > 0.50$ is complicated by the fact that the quantity $k = \Delta\tau_2/\Delta\tau_1$ becomes insensitive to the changes of ε and p. Therefore, we made use of such characteristics of the temperature field that contain quantities accessible to measurements with the smallest errors [4].

Equation (1) enabled us to construct a table of the dependence $\varepsilon/y = f(\varepsilon, \Theta)$. The graphs of this dependence are given in Fig. 1. It is evident from the graphs that for large values of Θ and small ε the ratio $(\varepsilon/y)\lim$ is independent of ε, i.e., $(\varepsilon/y)\lim = M$ will be constant for a given value $\Theta > 0.50$ and $\varepsilon < 0.7$ with an error of 1-3%.

In this case it is admissible to regard M as a single-valued function of Θ. The dependence $M = f(\Theta)$ is shown in Fig. 2. It follows from the notation in Eq. (1) that

$$\lambda = b\varepsilon\sqrt{a} = \frac{bh}{2\sqrt{\tau}} M. \tag{2}$$

Here the time τ is reckoned from the start of the contact of the heater with the sample. The numerical values of M for discrete values of Θ are given in Table 1.

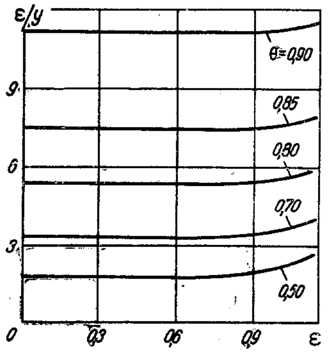

Fig. 1

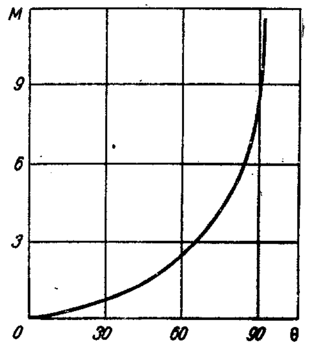

Fig. 2

Fig. 1. Graphs of the dependence $\varepsilon/y = f(\varepsilon, \Theta)$.

Fig. 2. Graph of the dependence $M = f(\Theta) \cdot \Theta \cdot 10^2$.

TABLE 1. Dependence $M = f(\Theta)$

θ	0,50	0,55	0,60	0,65	0,70	0,75	0,80	0,85	0,90
M	1,600	1,863	2,200	2,720	3,347	4,200	5,415	7,400	11,20

Applying formula (2) to two different time instants τ_1 and τ_2 we get

$$\lambda = \frac{bh}{2\sqrt{\Delta\tau}} M', \tag{3}$$

where

$$M' = \sqrt{M_2^2 - M_1^2} = \frac{\varepsilon}{\sqrt{p}}, \quad \Delta\tau = \tau_2 - \tau_1.$$

The values of M' corresponding to different values of Θ_1 and Θ_2 are given in Table 2.

For determining the coefficient of thermal conductivity in conditions $\varepsilon < 1$ we can use formulas that follow from the notation in Eq. (1):

$$a = \frac{h^2}{4 y_1^2 \tau_1} \tag{4}$$

or

$$a = \frac{h^2}{4 p \Delta\tau_1}. \tag{5}$$

In order to avoid the formation of large errors we need tables for determining the values of y_1 or we must choose p very rationally. In order to construct working tables for $y_1 = f(\tau_2/\tau_1)$ or $p = f(\Delta\tau_2/\Delta\tau_1)$ it is necessary to choose such values of Θ_i, for which the relative change of the functions y_1 and p would not differ much from the changes of the arguments $k' = \tau_2/\tau_1$ and $k = \Delta\tau_2/\Delta\tau_1$, i.e., the changes in $\Delta y/y$ and $\Delta p/p$ must not be very much smaller than the changes in $\Delta k'/k'$ and $\Delta k/k$, respectively.

TABLE 2. Dependence $M' = f(\Theta_1, \Theta_2)$

θ_2	θ_1							
	0,50	0,55	0,60	0,65	0,70	0,75	0,80	0,85
0,55	0,955							
0,60	1,51	1,17						
0,65	2,20	1,98	1,60					
0,70	2,94	2,78	2,52	1,95				
0,75	3,88	3,76	3,58	3,20	2,54			
0,80	5,18	5,09	4,95	4,68	4,25	3,42		
0,85	7,22	7,17	7,06	6,87	6,60	6,10	5,05	
0,90	11,08	11,05	10,98	10,86	10,69	10,38	9,80	8,40

The most suitable tables for determining the thermal diffusivity will be those constructed for the values $\Theta_1 < 0.40$ and $\Theta_2 > 0.65$. Otherwise, small errors in the determination of τ_2/τ_1 or $\Delta\tau_2/\Delta\tau_1$ lead to large errors in the determination of y_1 and p and, hence, of the thermal diffusivity a. On a semiautomatic device [3] observations can be made from the start of the contact of the heater with the sample and small time intervals can be determined quite accurately. In this case the thermophysical characteristics can be determined from formulas (2), (4) using the most suitable tables of $y_1 = f(\tau_2/\tau_1)$. The scheme of the laboratory equipment and the experimental technique remain the same as in the first method of two time–temperature intervals [3]. In order to satisfy the condition $\varepsilon < 0.7$ it is necessary to use heat receivers with large values of the heat activity b; this follows from the expression $\varepsilon = \lambda/b\sqrt{a}$. In the determination of t.p.c. of high heat-conductance samples of small thickness metallic heat receivers (lead, iron, copper) and heaters with large heat capacity should be used. It must be remembered that the length of the metallic heat receiver determined from the criterion $h \geq 4\sqrt{a_{\text{B}}\tau}$ must be large [3].

We especially note that the values of λ and a, obtained from formulas (2) and (4) or (3) and (5) for high heat-conductance materials are not the true values of the t.p.c. The effect of the contact lubrication has remained unclear and unaccounted for. These values of t.p.c. should be regarded as effective values $\overline{\lambda}$ and $\overline{a}$ for a multilayer system. In order to go from the effective values $\overline{\lambda}$ and $\overline{a}$ to the true values we can make use of the law of additivity of heat resistances:

$$R = \sum_{i=1}^{n} R_i. \tag{6}$$

This law holds for stationary heat regimes. However, it is shown in [5] that for $\varepsilon \leq 1$ and symmetric arrangement of the contact heat resistances, formula (6) is applicable with 1-2% error even for nonstationary regimes. In our case the law of additivity of heat resistances has the form

$$\frac{H}{\overline{\lambda}} = \frac{h}{\lambda} + R_0, \tag{7}$$

where

$$H = h + 2h_0,\ R_0 = \frac{2h_0}{\lambda_0}.$$

The values of R_0 for a given lubricant should be determined experimentally from a reference material with thermophysical properties and heat-transfer conditions close to the investigated materials. The heat receivers must be the same as in the determination of the t.p.c. of the investigated samples.

As follows from formula (7), the value of R_0 for the reference material A will be

$$R_0 = \frac{H_A}{\overline{\lambda}_A} - \frac{h_A}{\lambda_A} \simeq h_A\left(\frac{1}{\overline{\lambda}_A} - \frac{1}{\lambda_A}\right). \tag{8}$$

Here it has been assumed that $H_A = h_A + 2h_0 \simeq h_A$, since the thickness of the contact layer $2h_0$ must be very small, being a hundredth or a thousandth of a millimeter. Thus, for determining the thermal conductivity λ, first the effective value $\overline{\lambda}$ must be found from formula (2) or (3), then the quantity $p = (H/\overline{\lambda}) - R_0$, and finally, the value

TABLE 3. Results of Observations $\overline{\lambda}$ = (10.2 ± 0.2) W/m · deg

N/N_0, %	100	75	60	50	45	40	35	30	25
θ	0	0,25	0,40	0,50	0,55	0,60	0,65	0,70	0,75
τ, sec	0	5,4	15,5	30,0	38,7	56,0	79,2	115,5	196,0
$\sqrt{\tau}$				5,477	6,221	7,483	8,899	10,74	14,00
M				1,600	1,863	2,200	2,720	3,347	4,200
$\overline{\lambda}$				9,9	10,1	10,0	10,3	10,5	10,1

$$\lambda = h/p \tag{9}$$

must be computed according to formula (7). The coefficient of thermal diffusivity can be determined in the following way. The law of additivity of volumetric heat capacities has the form

$$V\bar{C} = \sum_{i=1}^{n} V_i C_i.$$

For our case this law will be

$$\bar{C}H = Ch + \delta, \text{ where } \delta = 2C_0 h_0.$$

Hence

$$C = \frac{\bar{C}H - \delta}{h}. \tag{10}$$

Knowing the heat capacity C we find

$$a = \lambda/C. \tag{11}$$

The quantity δ, taking account of the total effect of the contact lubrication on the results of observations, can be determined experimentally from the reference material A:

$$\delta = \bar{C}_A H_A - C_A h_A \simeq h_A (\bar{C}_A - C_A) = h_A \left(\frac{\bar{\lambda}_A}{\bar{a}_A} - \frac{\lambda_A}{a_A} \right), \tag{12}$$

since

$$H_A \simeq h_A.$$

We give an example of determination of t.p.c. of high heat conductance materials on thin samples.

The solid ceramic dielectric had thickness $h = 4.67 \cdot 10^{-3}$ m. The experiment was conducted from the start of the contact of the heater with the sample in an iron heat receiver with thermal activity $b = 14500$ W · sec$^{1/2}$/m · deg. Soap water was used for improving the thermal contacts. The observations and the results of the computations were as follows (see Table 3). For determining the coefficient of thermal diffusivity from formula (4) we can use (even though they are not the most suitable, they are available) tables [3] of the dependence $y_1 = f(\tau_2/\tau_1)$ for $N_1/N_0 = 0.75$, $N_2/N_0 = 0.50$ and for $N_1/N_0 = 0.50$; $N_2/N_0 = 0.25$. In the first case $\tau_1 = 5.4$ sec, $\tau_2 = 30.0$ sec, $\tau_2/\tau_1 = 5.55$, $y_1 = 0.517$; hence,

$$a = \frac{(4.67)^2 \cdot 10^{-6}}{4\,(0.517)^2 \cdot 5.4} = 378 \cdot 10^{-8} \text{ m}^2/\text{sec}$$

In the second case $\tau_1 = 30.0$ sec; $\tau_2 = 196.0$ sec; $\tau_2/\tau_1 = 6.53$; $y_1 = 0.220$; hence,

$$a = \frac{4.67^2 \cdot 10^{-6}}{4\,(0.22)^2 \cdot 30.0} = 375 \cdot 10^{-8} \text{ m}^2/\text{sec}$$

TABLE 4. Results of Analysis of the Observations $\bar{\lambda} = (10.4 \pm 0.3)$ W/m · deg

$\theta_1-\theta_2$	M'	$\Delta\tau$	$\sqrt{\Delta\tau}$	λ	$\Delta\lambda$
0,50—0,75	3,88	166,0	12,88	10,2	0,2
0,55—0,75	3,76	157,3	12,54	10,1	0,3
0,60—0,75	3,58	140,0	11,82	10,2	0,2
0,65—0,75	3,20	116,8	10,72	10,1	0,3
0,50—0,70	2,94	85,5	9,25	10,7	0,3
0,55—0,70	2,78	76,8	8,76	10,7	0,3
0,60—0,70	2,52	59,5	7,71	11,0	0,6
0,50—0,65	2,20	49,2	7,01	10,6	0,2
0,55—0,65	1,98	40,5	6,36	10,4	0,0
			Average	10,4	0,3

These same tables for $\varepsilon = f(\tau_2/\tau_1)$ give the values $\varepsilon_1 = 0.356$ and $\varepsilon_2 = 0.355$.

Then $\lambda = b\varepsilon\sqrt{a} = 14500 \cdot 0.356 \cdot 19.4 \cdot 10^{-4} = 10.1$ W/m · deg.

The proximity of the obtained value of λ to the mean value $\bar{\lambda} = 10.2$ W/m · deg obtained by an independent method is a tentative criterion of the reliability of the computed values of λ and a. The thermophysical characteristics λ and a can also be computed from formulas (3) and (5).

The observations are conveniently analyzed in accordance with the following scheme (see Table 4).

For determining the thermal diffusivity from formula (5) one can use the available tables [3] of $p = f(\Delta\tau_2/\Delta\tau_1)$ for $N_1/N_0 = 0.60$, $N_2/N_0 = 0.50$, $N_3/N_0 = 0.25$ or the table compiled from nodal points [3] for $N_1/N_0 = 0.50$, $N_2/N_0 = 0.40$, $N_3/N_0 = 0.30$.

In the first case $\Delta\tau_1 = 14.5$ sec; $\Delta\tau_2 = 180.5$ sec; $\Delta\tau_2/\Delta\tau_1 = 12.45$; $p = 0.0951$; hence

$$a = \frac{4{,}67^2 \cdot 10^{-6}}{4 \cdot 0{,}0951 \cdot 14{,}5} = 395 \cdot 10^{-8} \text{ m}^2/\text{sec}.$$

The same table gives $\varepsilon = 0.348$. Therefore $\lambda = b\varepsilon\sqrt{a} = 14500 \cdot 0.348 \cdot 19.9 \cdot 10^{-4} = 10.1$ W/m · deg.

In the second case $\Delta\tau_1 = 26.0$ sec; $\Delta\tau_2 = 85.5$ sec; $\Delta\tau_2/\Delta\tau_1 = 3.29$; $p = 0.0527$; $\varepsilon = 0.369$; hence

$$a = \frac{4{,}67^2 \cdot 10^{-6}}{4 \cdot 0{,}0527 \cdot 26{,}0} = 395 \cdot 10^{-8} \text{ m}^2/\text{sec},$$

$$\lambda = 14500 \cdot 0.369 \cdot 19{,}9 \cdot 10^{-4} = 10{,}6 \text{ W/m} \cdot \text{deg}.$$

Just as in the preceding case the obtained value of λ can be used for a preliminary check on the correctness of the determination of the effective values of the t.p.c. In the present example the effective values of the t.p.c. will be $\bar{\lambda} = 10.3$ W/m · deg, $\bar{a} = 386 \cdot 10^{-8}$ m²/sec, $\bar{c} = 2.67 \cdot 10^5$ J/m³ · deg. In order to go over from the effective values $\bar{\lambda}$ and $\bar{a}$ to the true values λ and a the calibration terms R_0 and σ must be determined from formulas (8) and (12) from the observations with the reference material. The following values of the t.p.c. were used for a reference sample of thickness $h = 4.65 \cdot 10^{-3}$ m:

$$\lambda_A = 21{,}0 \text{ W/m} \cdot \text{deg} \quad a_A = 600 \cdot 10^{-8} \text{ m}^2/\text{sec}.$$

$$\bar{\lambda}_A = 18{,}1 \text{ W/m} \cdot \text{deg} \quad \bar{a}_A = 545 \cdot 10^{-8} \text{ m}^2/\text{sec}.$$

In this case

$$R_0 = h_A\left(\frac{1}{\bar{\lambda}_A} - \frac{1}{\lambda_A}\right) = 4{,}65 \cdot 10^{-3}\left(\frac{1}{18{,}1} - \frac{1}{21{,}0}\right) = 35{,}4 \cdot 10^{-6} \text{ m}^2 \cdot \text{deg/W};$$

$$\delta = h_A\left(\frac{\bar{\lambda}_A}{\bar{a}_A} - \frac{\lambda_A}{a_A}\right) = 4{,}65 \cdot \left(\frac{18{,}1}{5{,}45} - \frac{21}{6{,}0}\right) \cdot 10^3 = -790 \text{ J/m}^3 \cdot \text{deg}.$$

Then

$$p = \frac{H}{\bar{\lambda}} - R_0 = \frac{4{,}67\cdot 10^{-3}}{10{,}3} - 0{,}0354\cdot 10^{-6} = 0{,}417\cdot 10^{-3}\ \text{m}^2\cdot\text{deg/W}\,,$$

and hence

$$\lambda = \frac{h}{p} = \frac{4{,}67\cdot 10^{-3}}{0{,}417\cdot 10^{-3}} = 11{,}2\ \ \text{W/m}\cdot\text{deg}$$

$$C = \frac{\bar{C}h - \delta}{h} = \frac{2{,}67\cdot 10^{6}\cdot 4{,}67\cdot 10^{-3} + 0{,}79\cdot 10^{3}}{4{,}67\cdot 10^{-3}} =$$

$$= \frac{13{,}24\cdot 10^{6}}{4{,}67} = 2{,}84\cdot 10^{6}\ \text{J/m}^3\cdot\text{deg}$$

$$a = \frac{\lambda}{C} = \frac{11{,}2}{2{,}84\cdot 10^{6}} = 395\cdot 10^{-8}\ \text{m}^2/\text{sec}\,.$$

In the method proposed here the values of the t.p.c. can be obtained repeatedly from a single experiment; this reduces the error in the determination of the t.p.c. When the conditions of the experiment indicated earlier, are satisfied, the error in the determination of the t.p.c. does not exceed 5%.

NOTATION

a, coefficient of thermal diffusivity; λ, coefficient of thermal conductivity; C, volumetric heat capacity; c, specific heat; $C = \lambda/a = c\rho$; ρ, density; b, thermal activity or heat-accumulating capacity; h, thickness of the investigated material; h_0, thickness of the lubricant contact layer; R, heat resistance; V, volume; θ, relative temperature; τ, time; N, galvanometer readings.

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